STRUCTURE OF A SWIRLED STREAM IN A CYLINDRICAL CHANNEL WITH UNIFORM INJECTION

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The results of an experimental investigation of the structure of the isothermal flow in a permeable pipe with stream swirling at the entrance are presented. A wide range of variation of the parameters and swirling laws is investigated.

The initial swirling of a gas stream is widely used in high-temperature power installations for the organization and improvement of the working process (vortical MHD generators, plasmatrons, etc.). For these conditions thermal protection by injection can prove to be the most effective in a number of cases. There are presently sufficiently detailed data in the literature on the structure of internal axial streams with injection [1, 2]. The relationships of the flow in pipes under the conditions of injection into a swirled stream are practically absent.

The present report is devoted to an experimental investigation of the local, integral, and turbulent characteristics of the isothermal swirled flow in an open cylindrical pipe 80 mm in diameter and 13.75 diameters long. A detailed description of the experimental installation is given in [3-5]. The stream was probed using thermoanemometers and pneumometric pressure pickups which were inserted into special openings made in the measurement section, which consists of a separate porous specimen with a jacket and an individual supply of the injected gas. In the investigations this section was mounted at distances of 3.06, 5.76, 8.46, and 12.51 diameters from the swirling sources. The method of conducting the experiments and treating the test data is similar to that for the flow of a swirled stream in an impermeable channel and is presented in detail in [6-11]. We note that the error in determining the velocities is ±5%, for the pulsation intensity it is ±10%, and for the correlations it is ±20%. The surface friction was determined on the basis of a transformation of the angular momentum and the longitudinal projection of the total momentum along the length of the channel, calculated from the known values of the local parameters (Euler's first and second theorems) [6]. The error of such an approach does not exceed ±10%. Finishing tests conducted under isothermal and nonisothermal conditions showed that the system of channel and measurement section satisfies the conditions of technical smoothness [4].

The initial swirling of the stream was accomplished by a vane swirler with a central body. The vanes of the swirler were laid out with respect to a power law $\tan\varphi = \tan\varphi_i (R/r)^n$. In the tests the main parameters of the swirler were varied in the limits of $\varphi_i = 15$-$60°$ and n = 1-3. In order to eliminate the penetration of the swirled stream into the injection cavity of the first section, an impermeable cylindrical channel with a relative length of about one diameter was mounted directly behind the swirler. The value of the coordinate $\bar{x}$ is given below with allowance for this section, while the value of the initial swirling intensity $\tilde{\Phi}_{*en}$ [6] corresponds to the cut of the impermeable section.

The tests were conducted under isothermal conditions with air injection into the swirled stream. An approximately constant value of the mass flow-rate density of the injected air along the length of the channel was maintained in all the experiments. The ranges of variation of the main parameters were $B_* = (1.17$-$10.06)\cdot 10^{-3}$, $b_x = 1.32$-4.62, $b_{x\delta} = 0.72$-3.74, $b_s = 0.77$-3.88, and $Re_{den} = (0.7$-$1.3)\cdot 10^5$.

Local and Integral Flow Parameters

An analysis shows that the structure of a swirled stream under the conditions of injection into it is determined by three main parameters: the swirling intensity Φ_*, the Reynolds number Re_d, and the injection parameter. Since the relative values of the local, integral, and turbulent characteristics of a swirled stream

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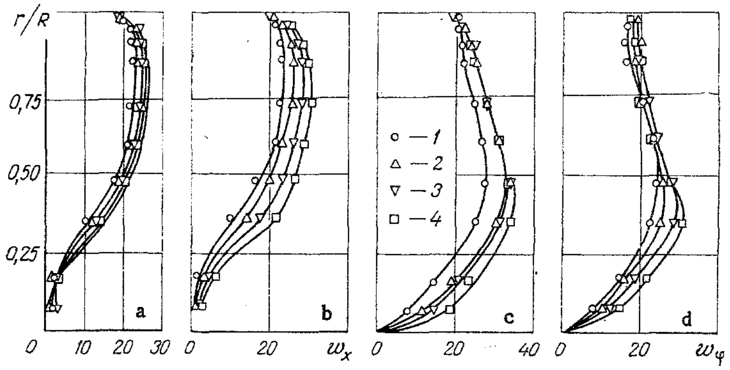

Fig. 1. Variation of axial (a, b) and rotational (c, d) velocity components along length of channel. Swirler with $\varphi_i = 45°$ and n = 3 ($\tilde{\Phi}_{*en} = 0.83$); $\bar{x} = 3.06$ (a, b); $\bar{x} = 8.46$ (c, d); 1) $B_* \cdot 10^3 = 0$; 2) 3.34; 3) 6.68; 4) 10.03. w_x, w_φ, m/sec.

are self-similar with respect to the Reynolds number Re_d [6], in this case the flow structure will be characterized by only two parameters.

The transformation of the axial and rotational velocity components along the length of the channel at different values of the injection parameter B_* is presented in Fig. 1. The slight increase in the axial velocity in the initial cross sections of the channel (Fig. 1a) in comparison with that for an impermeable channel is explained by the reduction of surface friction due to the injection. Later on (Fig. 1b), the formation of the w_x profile is conditioned by two main factors: an increase in the mass of gas moving in the channel, which leads to faster attenuation of the swirling, and the reduction of surface friction. For these reasons the maximum value of the axial velocity grows in absolute value and shifts into the axial region of the channel; the fullness of the w_x profile over a cross section of the channel gradually increases in the process.

As the analysis showed, in the investigated range of variation of the controlling parameters the relative values of the maximum axial and rotational velocities are uniquely determined by the swirling parameter Φ_*. The supply of additional mass (an increase in $\rho_0 w_{x0}$ and $\rho_0 w_{\Sigma 0}$) and the decrease in swirling intensity Φ_* (a decrease in $\rho_0 w_{x0}$ and $\rho_0 w_{\Sigma 0}$) approximately compensate for each other under the conditions of the experiment, so that the test data for impermeable and permeable pipes practically coincide with each other. Thus, for calculations in the region of $\Phi_* > 0.23$ one can use the equations [6]

$$\frac{\rho_0 w_{x0}}{\rho u} = 0.95 + \frac{1.8\Phi_*^{0.81}}{\mathrm{Re}_d^{0.12}}, \quad \frac{\rho_0 w_{\Sigma 0}}{\rho u} = 0.74 + \frac{5.75\Phi_*^{0.81}}{\mathrm{Re}_d^{0.12}}, \tag{1}$$

in which the values of ρu, Re_d, and Φ_* are determined with allowance for the supply of an additional mass of gas.

The rotational velocity in the initial cross sections of the channel grows slightly under the action of injection (Fig. 1c, d), which is also due to the reduction of surface friction. In this case the radius of the maximum value of the rotational velocity ($r_{\varphi\,max}$) shifts into the axial region (Fig. 1c, d). An analysis of the initial results of the experiments shows that this shift is due only to the decrease in the swirling intensity owing to injection. The numerical value of $\bar{r}_{\varphi\,max}$ is uniquely determined by the value of Φ_* and in the region of $\Phi_* > 0.44$ it can be calculated from the equation obtained for an impermeable pipe [6]:

$$\frac{r_{\varphi\max}}{R} = 0.19\,(1 + 2\Phi_*). \tag{2}$$

An effect analogous to diaphragming (narrowing) of the exit cross section of the channel develops under conditions of injection with relatively weak swirling ($0.18 \le \Phi_* \le 0.44$). In this case the calculating equation has the form

$$\frac{r_{\varphi\max}}{R} = 0.57\Phi_* + 0.1. \tag{3}$$

In Eqs. (2) and (3) the value of Φ_* is also determined with allowance for injection.

The integral swirling parameter Φ_* is the most important characteristic, and in a number of cases, as shown above, it uniquely determines the influence of injection on the flow structure. Treatment of the test data showed that for the investigated conditions, just as for an impermeable channel, the attenuation of the swirling obeys the exponential dependence

$$\Phi_* = \tilde{\Phi}_{*\,en}\exp(-p\bar{x}), \tag{4}$$

where $p = p_0(1 + 80B_*\tilde{\Phi}_{*en}^{0.76})$.

For flow in an impermeable channel the exponent is obtained on the basis of an analysis of the test data of [6] and is determined from the equations

$$\begin{gathered} \bar{x} < \bar{x}_1, \quad \Phi_* = \tilde{\Phi}_{*\,en}\exp(-p_{01}\bar{x}), \\ \bar{x} > \bar{x}_1, \quad \Phi_* = \tilde{\Phi}_{*\,en}\exp[(p_{02} - p_{01})\,\bar{x}_1 - p_{02}\,\bar{x}], \\ \bar{x}_1 = -4.7\tilde{\Phi}_{*\,en}^2 + 14.4\,\tilde{\Phi}_{*\,en} + 9, \\ p_{01}\cdot 10^2 = -0.47\tilde{\Phi}_{*\,en}^2 + 1.44\tilde{\Phi}_{*\,en} + 1.75, \quad p_{02}\cdot 10^2 = 1.9. \end{gathered} \tag{4a}$$

Just as for an impermeable channel, for injection into a swirled stream the connection between the swirling parameters Φ_* and Φ, which is used to solve the integral momentum equations [7], has the form of a power-law equation,

$$\Phi = B\Phi_*^b, \tag{5}$$

where the values of B and b are determined by the injection parameter b_x:

$$B = B_0(1 - 0.08b_x), \quad b = b_0(1 - 0.06b_x). \tag{6}$$

The quantities B_0 and b_0 represent the values of the parameters in the absence of injection and with the analogous geometrical conditions ($\bar{d}$, l/d) and are determined from the data presented in [8].

One of the important properties of internal flow swirling discovered in the present investigation is the universal one-to-one connection between the swirling parameter Φ_* and the limiting angle of stream swirling at the surface of the channel, tan φ_w, which characterizes the relative curvature of the streamlines of the translational – rotational motion. For the flow of a swirled stream in an impermeable and a permeable cylindrical channel this connection has the form of a power-law equation,

$$\operatorname{tg}\varphi_w = 1.2\Phi_*^{0.784}, \tag{7}$$

which determines the very important connection between the local and integral parameters of an internal swirled stream.

With a known law of swirling attenuation and the connection (7) one can also find an expression for the relative curvature of the streamlines of the translational – rotational motion near the surface of the channel from the equation

$$\frac{\eta_w}{R} = \frac{(1 + \operatorname{ctg}^2\varphi_w)^{1.5}}{\left[1 + \operatorname{ctg}^2\varphi_w + \operatorname{ctg}^4\varphi_w\left(\frac{p}{2}\right)^2\right]^{0.5}}, \tag{8}$$

where $p = 0.784p_{01}$ for $\bar{x} < \bar{x}_1$ and $p = 0.784p_{02}$ for $\bar{x} > \bar{x}_2$.

An analysis of Eqs. (4a) shows that the condition tan $\varphi_w \gg p/2$ is satisfied in a wide range of variation of the controlling parameters. This means that for practical calculations one can use the simplified equation

$$\frac{\eta_w}{R} = 1 + \operatorname{ctg}^2\varphi_w. \tag{8a}$$

Region of Boundary Flow

For "pure" circular motion of the stream the sum of the quantities characterizing the generation and dissipation of turbulence energy is approximately equal to zero near the surface of the channel [9]. This means that the relations determined by the Prandtl hypothesis are satisfied here. The translational – rotational motion of the stream can also be reduced to an analysis of a "purely" circular stream by representing it as a collection of separate rotations about an instantaneous velocity center, which is determined by Eq. (8a). In this case the expression for the Prandtl hypothesis in the coordinate system ξ, ζ, η (see Sec. 3) will have the form

$$\tau_{\eta\xi} = \rho l_*^2\left(\frac{\partial w_\Sigma}{\partial \eta}\right)^2. \tag{9}$$

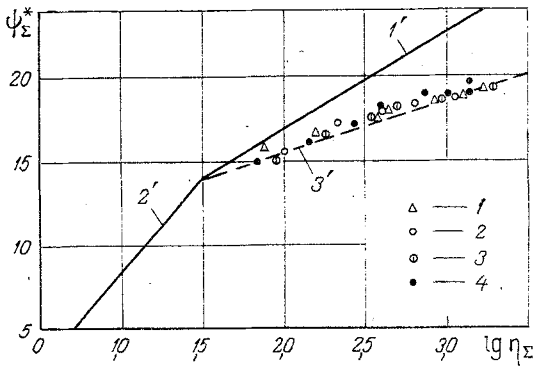

Fig. 2. Universal profile of total stream velocity in region of boundary flow (n = 3 for 1-4): 1) $\varphi_1 = 60°$, $B_* \cdot 10^3 = 9.95$, $b_{x\delta} = 2.19$, $\Phi_* = 0.57$; 2) 45°, 3.34, 0.83, 0.57; 3) 45°, 6.68, 1.72, 0.58; 4) 45°, 10.03, 2.37, 0.53; 1', 2') $b_{x\delta} = 0$, $\Phi_* = 0$; 3') $b_{x\delta} = 0$, $\Phi_* = 0.55$.

Since near the surface of the channel the stream swirling angle varies quite insignificantly, under assumptions analogous to those given in [10] we can obtain from Eq. (9) the expression

$$\varphi_\Sigma^* = \frac{1}{\varkappa_*} \ln \eta_\Sigma + \text{const}, \tag{10}$$

from which it follows that outside the region of the laminar sublayer in the coordinates φ_Σ^*, η_Σ the variation in the total velocity near the surface of the channel must obey a logarithmic law. The region where the influence of the wall on the stream structure is manifested and the logarithmic law is satisfied will henceforth be called the region of boundary flow. This region is located almost up to the radius of the maximum value of the total velocity.

The first step was the investigation of the laws of boundary flow in an impermeable cylindrical channel with the same geometrical size. As a result, we obtained the equation

$$\varphi_\Sigma = [5.5 + 4.65\,(\Phi_* - 0.07)^{0.26}] + [5.75 - 3.36\,(\Phi_* - 0.07)^{0.3}] \ln \eta_\Sigma, \tag{11}$$

which can be used for a wider range of variation of the controlling parameters than that given in the authors' report [6].

An analysis of the test data obtained for the permeable channel allowed us to conclude that in the coordinate system φ_Σ^*, η_Σ the total velocity distribution in the region of boundary flow practically coincides with the calculated data obtained from Eq. (11). This conclusion is confirmed by the partial results of the investigation presented in Fig. 2. The latter is in agreement with results obtained for axial flow at a permeable plate [10].

Pulsation Intensity

The microstructure of the swirled stream was determined in the coordinate system ξ, ζ, η connected with a helical streamline [11], which is due to the methodological properties of the performance of the experiments using a thermoanemometer. Here the ξ coordinate coincides with the direction of the total velocity while ζ and η are perpendicular to it (η is the radial coordinate).

The results of a determination of the three components of the pulsation intensity are presented in Fig. 3. Just as in axial flow at a plate, the transverse flow from the channel wall promotes an increase in the longitudinal and transverse components of the pulsations near the surface of the channel. The radial component grows over almost the entire cross section of the channel.

The overall character of the radial variation of the components of the pulsation intensity (a curve with a minimum) is analogous to that for swirled flow in an impermeable channel [11] and indicates the simultaneous existence of flow regions with the active and conservative action of centrifugal mass forces. This means that the corresponding terms in the equation of turbulent energy balance must change sign at the point of the pulsation minimum.

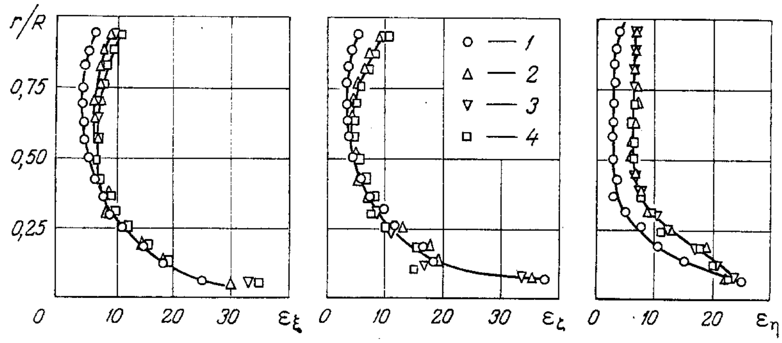

Fig. 3. Intensity of longitudinal (ε_ξ), transverse (ε_ζ), and radial (ε_η) pulsations in channel. Swirler with $\varphi_i = 45°$, n = 3, $\bar{x} = 8.46$; 1) $B_* = 0$; 2) 0.0033; 3) 0.0066; 4) 0.01.

Friction Law and Form Parameters of Stream

The test data on surface friction were generalized in the form of a relative law using the principle of superpositions of separate actions (injection and swirling):

$$\left(\frac{c_x}{c_{0x}}\right)_{\mathrm{Re}^{**}} = \Psi_\varphi \Psi_{\mathrm{inj}}. \tag{12}$$

Here it was assumed that the influence of swirling on the surface friction is determined by the equation

$$\Psi_\varphi = -2.16\Phi_*^2 + 3.42\Phi_* + 0.6; \quad \Phi_* > 0.13, \tag{13}$$

which was obtained in [8] for flow in an impermeable channel in a wide range of variation of the controlling parameters.

The results of a determination of the relative function Ψ_{inj}, which allows for the influence of injection on the surface friction, are presented in Fig. 4 and are satisfactorily generalized by the equation

$$\Psi_{\mathrm{inj}} = \left(\frac{1 - 0.11 b_x}{1 + 0.11 b_x}\right)^2. \tag{14}$$

Thus, the friction law for the isothermal flow of a swirled stream in a pipe under conditions of uniform injection will have the form

$$\frac{c_x}{2} = \frac{c_{0x}}{2}\left(-2.16\Phi_*^2 + 3.42\Phi_* + 0.6\right)\left(\frac{1 - 0.11 b_x}{1 + 0.11 b_x}\right)^2 \tag{15}$$

and can be used to solve the integral momentum equations obtained by differentiation of the differential equations of motion over the entire cross section of the channel (see the approach presented in [8]). Here $c_{0x}/2$ is the "standard" friction law for quasi-isothermal nongradient flow at a plate [1].

Because of the very weak variation in the angle of stream swirling near the surface of the channel [6], from Eq. (15) we can determine the tangential frictional stresses $\tau_{\varphi w}$ and $\tau_{\Sigma w}$ from the equations

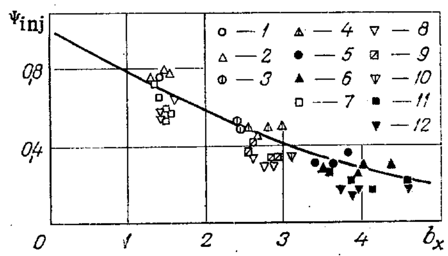

Fig. 4. Relative injection function for stream swirling in a pipe: 1) $B_* \cdot 10^3 = 3.31$; 2) 3.34; 3) 6.63; swirler 4) $B_* \cdot 10^3 = 6.68$; swirler 5) $B_* \cdot 10^3 = 9.95$, $\varphi_i = 60°$, n = 3; 6) 10.03, 45°, and 3, respectively; 7) $B_* \cdot 10^3 = 3.33$; 8) 3.36; 9) 6.69; swirler 10) $B_* \cdot 10^3 = 6.71$; swirler 11) $B_* \cdot 10^3 = 10.0$, $\varphi_i = 45°$, n = 1; 12) 10.06, 15°, and 3.

$$\tau_{\varphi w} = \tau_{xw}\,\mathrm{tg}\,\varphi_w, \quad \tau_{\Sigma w} = \tau_{xw}\sqrt{1+\mathrm{tg}^2\varphi_w},$$

and also the corresponding coefficients of friction. It should be noted that the approximate equality $c_x/2 \approx c_\varphi/2$ occurs in this case, since the ratio $\Gamma_0/(w_{x0}R\tan\varphi_W)$ is practically equal to unity.

At the same time, we found the relative injection functions $\Psi_{\text{inj}\,\delta}$ and $\varepsilon_{\text{inj}\,s}$, while the results of the generalization were determined by the equations

$$\Psi_{\text{inj}\,\delta} = \left(\frac{1-0.13b_{x\delta}}{1+0.13b_{x\delta}}\right)^2, \quad \varepsilon_{\text{inj}\,s} = \left(\frac{1-0.15b_s}{1+0.15b_s}\right)^2. \tag{16}$$

The influence of injection on the form parameters of the swirled flow was found on the basis of the principle of superpositions of separate actions (injection and swirling):

$$H_x = H_0 H_x^\varphi H_x^{\text{inj}}, \quad H_{x\varphi} = H_0 H_{x\varphi}^\varphi H_{x\varphi}^{\text{inj}}. \tag{17}$$

Here the values of H_x^φ and $H_{x\varphi}^\varphi$ in the absence of injection were determined on the basis of the equations presented in [8] for flow in an impermeable channel. The calculating equations determining the influence of injection on the form parameters have the form

$$H_x^{\text{inj}} = 1 + 0.05b_x, \; H_{x\varphi}^{\text{inj}} = 1 + 0.066b_x \tag{18}$$

and can be used to solve the integral momentum equations.

The results presented in this article reveal the physical bases of the process of flow of a swirled stream in a permeable cylindrical channel and comprise a basis for the development of an integral method of calculating such flows.

NOTATION

$B_* = (\rho v)_w/(\rho u)_{en}$, permeability parameter; $b_x = (\rho v)_w/\rho_0 w_{x0}$; $b_{x\delta} = [(\rho v)_w/\rho_0 w_{x0}](2/c_{0\delta})$; $b_3 = [(\rho v)_w/\rho_0 w_{x0}](2/c_{0x})$, injection parameters; $c_x/2 = \tau_{xw}/\rho_0 w_{x0}^2$; $c_{0S}/-2 = 0.0288\,\mathrm{Re}^{-0.2}$; $c_\varphi/2 = \tau_{\varphi w}R/\rho_0 w_{x0}\Gamma_0$; $c_{0\delta}/2 = 0.0128\,\mathrm{Re}_\delta^{**-0.25}$, coefficients of friction; $\bar{d} = d_g/2R$, relative diameter of conical exit diaphragm; h_x, $h_{x\varphi}$, form parameters of stream; l/d, relative length of channel; n, index of intensity of twisting of swirler vanes; R, radius of channel; $\mathrm{Re}^{**} = \mu_0 w_{x0}\delta_x^{**}/\mu$; $\mathrm{Re}_\delta^{**} = \rho_0 w_{x0}\delta^{**}/\mu$; $\mathrm{Re}_d = \rho u 2R/\mu$; $\mathrm{Re}_{den} = (\rho u)_{en}2R/\mu$; $\mathrm{Re}_s = \rho_0 w_{\Sigma 0} x_s/\mu$, Reynolds numbers; r, φ, x, cylindrical coordinates; $\bar{x} = x/2R$, relative coordinate; x_s, length of helical line along surface of channel; $\Phi = M/K_\Sigma R$; $\Phi_* = M/K_1R$, present intensity of swirling; M, K_1, angular and longitudinal momenta; K_Σ, longitudinal projection of total momentum of stream; Φ_{*en}, mean true intensity of swirling at exit from swirler; w, velocity; Γ_0, circulation of rotational velocity at outer limit of region of boundary flow; δ_x^{**}, δ^{**}, momentum thickness calculated over entire channel cross section and in region of boundary flow, respectively; ρ, density; φ, φ_i, present and initial (at r = R) angles of vane twisting; $\varphi_\Sigma^* = (2\rho/(\rho v)_w)[\sqrt{(\tau_{\Sigma w}/\rho) + (w_\Sigma/\rho)(\rho v)_w} - \sqrt{\tau_{\Sigma w}/\rho}]$, frictional stress at channel wall; $\varepsilon_i = \sqrt{w_i'}/w_\Sigma$ $(i = \xi, \zeta, \eta)$; $\eta_\Sigma = (y/\nu)\sqrt{\tau_{\Sigma w}}/d$; y, distance from wall. Indices: en, entrance; Σ, total; 0, maximum conditions; δ, conditions at outer limit of region of boundary flow.

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MATHEMATICAL ANALYSIS OF WHIRLED TURBULENT FLOW THROUGH A PIPE

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UDC 532.542.4

The effect of rotation of the stream on the development of turbulent flow in a pipe is analyzed by a numerical method. Calculated distributions of average turbulence velocity and energy are compared with experimental data.

Study of whirled turbulent flow is very important. Owing to the tremendous complexity of such a flow, however, it has so far been studied less extensively than similar flow without whirling. This applies especially to flow through pipes. Here will be presented the results of numerical calculations pertaining to whirled turbulent flow through a cylindrical pipe, calculations based on the two-parametric k – ε and k – W models of turbulence [1]. Various authors have used these models earlier for calculating the flow in boundary layers, in free or bounded jets, and through channels of intricate shapes. They compared the theoretical and experimental data on the basis of average flow characteristics (velocity profiles, size and location of the recirculation zone, etc.). In [2], e.g., a comparison is shown between calculated and measured profiles of average velocity along an annular channel. This is partly attributable to the fact that published experimental data on whirled flow are incomplete in terms of turbulence characteristics. For this reason, we have selected for comparison the data in [3] containing not only the profiles of the components of average velocity and the pressure distributions along the pipe wall as well as along the pipe axis, but also data on the distribution of and the correlation between the intensities of the three components of velocity fluctuations in the stream.

The system of equations describing a steady turbulent motion of an incompressible fluid through a pipe, under the assumption of a rotationally symmetric flow without external body forces acting and with constant molecular transfer coefficients, can be written in cylindrical coordinates as

$$
\begin{gathered}
\frac{\partial v_z}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(rv_r)=0,\\
\frac{\partial v_z v_r}{\partial z}+\frac{1}{r}\frac{\partial r v_r^2}{\partial r}=\frac{\partial}{\partial z}\left[(\nu+\nu_t)\frac{\partial v_r}{\partial z}\right]+\frac{2}{r}\frac{\partial}{\partial r}\left[r(\nu+\nu_t)\frac{\partial v_r}{\partial r}\right]+\frac{\partial}{\partial z}\left[(\nu+\nu_t)\frac{\partial v_z}{\partial r}\right]-2(\nu+\nu_t)\frac{v_r}{r^2}+\frac{v_\theta^2}{r}-\frac{1}{\rho}\frac{\partial p}{\partial r},\\
\frac{\partial v_z^2}{\partial z}+\frac{1}{r}\frac{\partial r v_r v_z}{\partial r}=2\frac{\partial}{\partial z}\left[(\nu+\nu_t)\frac{\partial v_z}{\partial z}\right]+\frac{1}{r}\frac{\partial}{\partial r}\left[r(\nu+\nu_t)\frac{\partial v_r}{\partial z}\right]-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left[r(\nu+\nu_t)\frac{\partial v_z}{\partial r}\right], \qquad (1)\\
\frac{\partial r v_z v_\theta}{\partial z}+\frac{1}{r}\frac{\partial r^2 v_r v_\theta}{\partial r}=\frac{\partial}{\partial z}\left[(\nu+\nu_t)\frac{\partial r v_\theta}{\partial z}\right]+\frac{1}{r}\frac{\partial}{\partial r}\left[r(\nu+\nu_t)\frac{\partial r v_\theta}{\partial r}\right]-\frac{2}{r}\frac{\partial}{\partial r}\left[(\nu+\nu_t)rv_\theta\right],
\end{gathered}
$$

where v_z, v_r, and v_θ are time-averaged components of velocity.

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